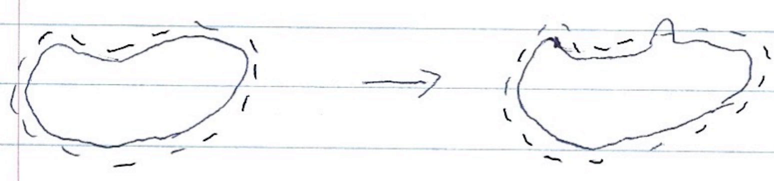
Jackson extra

Prove that the charge density induced on a conductor is the normal derivative of the potential (eq. 2.5)

We first argue that the surface charge density is given by the difference in normal components of $\vec{E}$:

$$(\vec{E}_2 - \vec{E}_1) \cdot \hat{n} = \sigma / \varepsilon_0 \qquad \text{(eq. 1.22)}.$$

Consider surface bounded by dashed lines, we draw a surface just bounded below it with solid lines. Then we introduce an infinitesmal "pimple" on the inner surface so that it goes slightly above the dashed surface.



It's clear that by gauss's law, equating the change in normal E field component with change in charge density, we obtain (eq. 1.22).

Then $(\vec{E}_{above} - \vec{E}_{below}) \cdot \hat{n} = \sigma / \varepsilon_0$,

$$-\vec{\nabla}(\Phi_{above} - \Phi_{below}) \cdot \hat{n} = \sigma / \varepsilon_0,$$

$$\boxed{-\vec{\nabla}\, \Phi_{conductor} \cdot \hat{n} = \sigma / \varepsilon_0}$$

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